

the "Sheffer stroke") are complete. By an operation we mean a (single-valued) function whose domain is S^n for some set S and positive integer n . A set F of operations on S is complete if any operation f on S (of any number of arguments) can be constructed from F by composition (substitution) and identification of variables.

The first three chapters, which are introductory, include, among other things, a discussion of how constructing f from F corresponds to constructing a network "realizing" f from elements (primitive networks) which realize the elements of F . Chapter IV is preparatory to Chapter V, where the first significant theorem appears. This is to the effect that sum modulo p (p a prime), product modulo p , and the constant functions are complete. Alternatively, every n -ary operation on $0, 1, 2, \dots, p-1$ is representable by a polynomial in n variables over the field of integers, modulo p . The author fails to note, however, that for any finite field, any operation on the p^n field elements is representable by a polynomial over the field. As a matter of fact, essentially the same argument the author gives for p elements is applicable to the more general situation.

Chapter VI deals with a theorem of Webb to the effect that the binary operation W defined over $0, 1, \dots, m-1$ by $W(x, y) = 0$ if $x \neq y$ and by $W(x, y) = x + 1, \text{ mod } m$, if $x = y$ is complete. The author gives a formulation of the theorem which does not make use of the additive structure on the set, and gives a proof of it.

The last chapter (VII) generalizes a theorem of E. L. Post to the effect that if R is a permutation of E , the integers mod m , then the pair of operations \otimes_R, P_R is complete, where $R(i) \otimes_R R(j) = R(\min i, j)$ and $P_R(R(i)) = R(i + 1)$, for all $i, j \in E$.

The following misprints were discovered:

- p. 39 line 10, $E = (0, 1 \dots, n-1)$ should read $E = (0, 1, \dots, p-1)$;
- p. 40 (63), read $A_{r,q}$ for $A_{r,q}$;
- p. 40 line 6 from bottom, read $M_{r,i}$ for $M_{r,i}$;
- p. 51 line 2 from bottom, read p^n terms for p terms;
- p. 55, 56; each recurrence of bg should read b_a ;
- p. 59 last line, $a = \delta ab$ should read $q = \delta ab$.

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38[I].—A. O. GELFOND, *Differenzenrechnung*, Deutscher Verlag, Berlin, 1958, viii + 336 p., 23 cm. Price DM 40.

This is a translation of the Russian edition (1952) which is a revised and extended version of an earlier book (1936). It is mainly concerned with problems in the complex domain, and some material, traditional in courses on Finite Differences, is omitted. The techniques used are those of classical analysis. There are occasional sets of problems, and some very interesting worked examples.

The book is divided into three large chapters (1, 2, 5) and two smaller ones (3, 4). Chapter One deals with the problem of interpolation ("construct an (ap-

proximate) expression for a function, given its values at a discrete set of points”), and includes an account of the Chebyshev theory.

Chapter Two deals with the Newton Series, first for equally spaced nodes, then for more general cases. The chapter concludes with applications of interpolation-theoretic methods to number-theoretic problems, in particular, to a proof of the theorem that α and $\beta = e^\alpha$ cannot both be algebraic, except for $\alpha = 0$.

The early part of Chapter Five is concerned with conventional material, including, for example, Ostrowski's proof of Hölder's result that $\Gamma(z)$ does not satisfy an algebraic differential equation; the latter part is concerned with work of the author (1951) on linear differential equations of infinite order, with constant coefficients.

Chapter Three is concerned with earlier (1937) research of the author on the construction of (entire) functions given their values at a series of points a_n , $a_n \rightarrow \infty$, and with related problems, e.g., the uniqueness of such functions.

Chapter Four contains standard material on the Summation Problem and the theory of Bernoulli numbers and polynomials; it includes, e.g., a proper account of the Euler-MacLaurin Summation Formula.

The book is clearly and precisely written. It can be recommended as an excellent source for many of the basic theorems in numerical analysis, and is a very suitable complement to such books as Natanson [1], which is largely concerned with the real domain.

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1. I. P. NATANSON, *Konstruktive Funktionentheorie*, Akad. Verlag, Berlin, 1955 [MTAC Review 9, v. 13, 1959, p. 64-67.]

39[I, X].—J. KUNTZMANN, *Méthodes Numériques*, Dunod, Paris, 1959, xvi + 252 p., 25 cm. Price NF 36.00.

The author (who is a professor of applied mathematics of the Faculty of Sciences at Grenoble) admits his concern over the lack of a suitable textbook in numerical mathematics written in French. Rather than translate a foreign (to him) work, he decides to write a new book.

For various reasons he decides to limit his book almost exclusively to interpolation. The usual interpolation formulas (Newton-Gregory, Stirling, Gauss, Bessel, Everett, and Lagrange) are included for equally-spaced abscissas and also for divided differences as appropriate.

For the most part, approximation by the standard sets of polynomials (Legendre, Chebyshev, etc.) is avoided, but Bernoulli polynomials and Bernoulli numbers are discussed.

More general formulas for which the given data might be either values of the function or values of certain derivatives are discussed. Numerical integration is avoided, but interpolation for functions of two or more variables as well as of a complex variable is included. The last two chapters deal with the theory of interpolation for linear sums of special functions (exponentials, trigonometric sums, etc.)

Since the book was written to fulfill a need in France, and since there is no co.